[5 PTS] Find all zeros of $f(x) = 3x^3 - 2x^2 + 17x + 6$.

ANSWER: $-\frac{1}{3}$, $\frac{1}{2} \pm \frac{\sqrt{23}}{2}i$

$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & -2 & 17 & 6 \\ & & -1 & 1 & -6 \\ \hline & 3 & -3 & 18 & \boxed{0} \end{array}$$

$f(x) = (x+\frac{1}{3})(3x^2-3x+18)$

$= 3(x+\frac{1}{3})(x^2-x+6)$

$x = \frac{1 \pm \sqrt{1-24}}{2} = \frac{1 \pm \sqrt{-23}}{2} = \frac{1 \pm \sqrt{23}\,i}{2}$

[3 PTS] Write the quotient in standard form $\frac{3i}{(4-5i)^2}$.

ANSWER: $-\frac{120}{1681} - \frac{27}{1681}i$

$\frac{3i}{16-40i+25i^2}$

$= \frac{3i}{-9-40i} \cdot \frac{-9+40i}{-9+40i}$

$= \frac{-27i+120i^2}{81-1600i^2} = \frac{-120-27i}{1681}$

[2 PTS] List all possible rational zeros of $f(x) = 6x^5 + 9x^3 - 12x^2 - 24x + 4$.
<u>You do NOT need to find any actual zeros.</u>

ANSWER: $\pm 1, \pm 2, \pm 4, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{1}{6}$

$\pm \frac{1, 2, 4}{1, 2, 3, 6}$

2

$-\frac{1}{2}$ FOR EACH ERROR OR MISSING #

ADDITIONAL QUESTIONS ON THE OTHER SIDE ➡

[2 PTS] Check if $x = -2$ is a lower bound of the real zeros of $f(x) = 2x^3 + 3x^2 - 3$. **ANSWER:** ½ YES (YES or NO)

In words, give a very brief reason for your answer.

$$\begin{array}{r|rrrr} -2 & 2 & 3 & 0 & -3 \\ & & -4 & 2 & -4 \\ \hline & 2 & -1 & 2 & -7 \end{array}$$

REASON: ½ SIGNS ALTERNATE

[2 PTS] Simplify i^{75} and write in standard form. **ANSWER:** $-i$

$$\begin{array}{r} 18 \\ 4\overline{)75} \\ \underline{4} \\ 35 \\ \underline{32} \\ 3 \end{array}$$

$i^{75} = i^3 = -i$

[4 PTS] Find a polynomial function with real coefficients that has 2 and $5+i$ as zeros. **ANSWER:** $x^3 - 12x^2 + 46x - 52$

$(x-2)(x-(5+i))(x-(5-i))$

$= (x-2)((x-5)-i)((x-5)+i)$

$= (x-2)((x-5)^2 - i^2)$

$= (x-2)(x^2 - 10x + 25 + 1)$

$= (x-2)(x^2 - 10x + 26)$ 2

$= x^3 - 10x^2 + 26x$

$\quad - 2x^2 + 20x - 52$

$= x^3 - 12x^2 + 46x - 52$

[2 PTS] Use Descartes' Rule of Signs to determine the possible numbers of positive and negative zeros of $f(x) = x^5 - x^4 + 4x^3 + 15x^2 + 58x - 40$. **ANSWER:** positive zeros: 3 OR 1 negative zeros: 2 OR 0

$f(-x) = -x^5 - x^4 - 4x^3 + 15x^2 - 58x - 40$